

## Experimental Investigation of $V-A$ in Leptonic Lambda Decay

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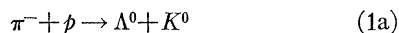
(Received 24 February 1964)

Twenty-two  $\Lambda \rightarrow p^+ + e^- + \bar{\nu}$  events were observed from  $K^0\Lambda^0$  production in hydrogen, for which the lambdas are highly polarized and three events from  $K^0\Sigma^0$  production. The 22 polarized events were studied as to the space-time properties of the interaction currents. Assuming vector and axial vector currents, a likelihood fit to the data was used to determine a best value for the ratio and sign of the coupling strengths  $k = C_A/C_V$ . The data was most consistent with  $C_V = -C_A$ . Higher order terms in the coupling form factors were looked at, but the statistical errors were too large to permit any significant statements concerning them. The branching ratios for  $\Lambda_e$  and  $\Lambda_\mu$  decays are reported and are consistent with  $V-A$  and muon-electron universality. Finally, the values obtained for  $k$  and the  $\Lambda_e$  branching ratio are compared to the theory of  $N$ . Cabibbo and seen to be in remarkable agreement.

THE universal-Fermi-interaction  $V-A$  theory has successfully explained most leptonic processes, but predicts hyperon leptonic decay rates an order of magnitude higher than observed experimentally.<sup>1</sup> It is of interest, therefore, to investigate the nature of the interactions for such decays to learn if  $V$  and  $A$  are indeed the currents, what their relative strength and sign are, and whether or not muon-electron universality persists.

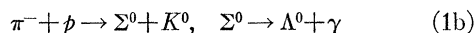
The lambda hyperon is particularly useful for such a study because each event can be completely determined kinematically, and highly polarized lambdas are available.

We report here an experimental study of the reaction  $\Lambda^0 \rightarrow p^+ + e^- + \bar{\nu}$  from a sample of highly polarized lambdas produced in the reaction



in the Alvarez 72-in. hydrogen bubble chamber.

For purposes not requiring polarization, events are also included from the reaction



(but only when both  $V$ 's are seen). Results on the lambda beta and muonic branching ratios are also presented.

The beam momenta range from 1030 to 1325 MeV/ $c$ , with the majority of the pictures at 1030 (the  $K^0\Sigma^0$  threshold).

All  $V^0$  events were measured either on a digitized microscope or projector. The events were then fitted with the PANG-KICK kinematic computer programs. Events accepted as normal  $K^0 \rightarrow \pi^- + \pi^+$  and  $\Lambda^0 \rightarrow p^+ + \pi^-$  decays were required to have a 3-constraint  $\chi^2$  less than 25. If the  $\chi^2$  was greater than 25 it was considered as a possible leptonic lambda decay candidate. A Monte Carlo program showed that 7.5% of the

lambda beta decays and 16% of the lambda muonic decays would remain undetected by this cutoff.

The separation of the leptonic lambda decays from all other "failing" events constituted the core of this experiment. The separation was made on the basis of a scanning table observation and/or by means of a diagnostic computer calculation.

"Failing" events easily eliminated by a scanning table observation were: (a) electron pairs; (b) Dalitz pairs (from the neutral decay mode of  $K^0$  or  $\Lambda^0$ ); (c) three-body and other failing  $K^0$  decays; (d)  $\pi^- \rightarrow \mu$  or  $\mu \rightarrow e$  decays displayed in a  $V$ -shaped configuration; and (e)  $\Lambda^0 \rightarrow p^+ + \pi^-$ , the  $\pi^-$  positively identified because it either (i) stopped in the chamber, (ii) decayed into a muon, or (iii) underwent a nuclear scatter or charge exchange in flight. (a) through (d) were easily eliminated because no proton was present, which was easily identified by ionization and curvature in this experiment.

The hypotheses tested by the diagnostic computer calculation were: (a)  $\Lambda \rightarrow \pi^- +$  (setup<sup>2</sup>  $p^+$ ) to test for a proton scatter; (b)  $\Lambda \rightarrow p^+ +$  (setup  $\pi^-$ ) to test for a  $\pi^-$  scatter or decay; (c)  $\Lambda \rightarrow p^+ + e^- + \bar{\nu}$ ; (d)  $\Lambda \rightarrow p^+ + \mu^- + \bar{\nu}$ ; (e)  $\Lambda \rightarrow p^+ + \pi^- + \gamma$ ; (f) (setup  $\Lambda$ )  $\rightarrow p^+ + \pi^-$  to test for a badly measured neutral track or a  $\Lambda$  scatter before decay; (g) coplanarity test of the measured tracks to help in deciding among (a) through (f).

Requirements for acceptance as hypothesis (a), or (b) were a goodness of fit, consistency between the measured track and the computer setup track within a scatter hypothesis, and a visual observation on the scanning table of the kink in the track. Eight events satisfied only the first two requirements, three in category (a) and five in category (b). These numbers are consistent with the number of scatters one might expect within 5 mm of the decay vertex. Scatters this close would be hard to detect. Four of the eight also fit a leptonic lambda decay hypothesis. However, the chance of an actual leptonic lambda decay satisfying the first two requirements of hypothesis (a) or (b) is quite small.

<sup>2</sup> Set-up means that the event was fitted without any information from the setup track except a specification of its mass.

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<sup>1</sup> See, for example: R. P. Ely, G. Gidal, G. E. Kalmus, L. O. Oswald, W. M. Powell *et al.*, Phys. Rev. **131**, 868 (1963). W. E. Humphrey, J. Kirz, and A. H. Rosenfeld, Phys. Rev. Letters **6**, 478 (1961).

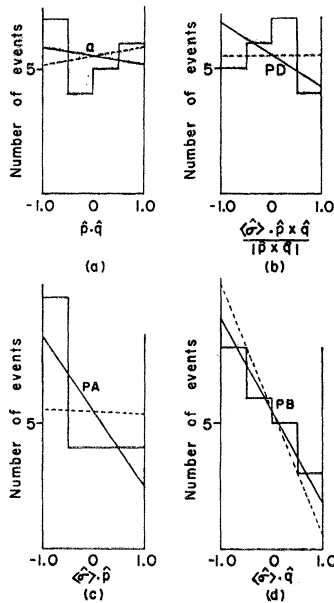


FIG. 1. Histogram plots for the angular distributions of Eq. (2) for 22 polarized  $\Lambda \rightarrow p + e^- + \bar{\nu}$  decays. The slopes (solid lines) are determined by the likelihood method for each distribution independent of the others. The values obtained are  $a = -0.06 \pm 0.34$ ,  $PD = -0.28 \pm 0.35$ ,  $PA = -0.55 \pm 0.31$ , and  $PB = -0.67 \pm 0.35$ . Assuming  $D=0$ , the over-all maximum likelihood for  $k = C_A/C_V$  is determined, constrained to Eq. (2). The slopes resulting from this value for  $k$  are represented by the dashed lines. According to a computed chi square, the probability of having obtained a worse fit is 70%.

It is believed, therefore, that even these four are probably actually scatters. Therefore, all eight events were accepted as close-in scatters.

Similar rigid requirements were demanded of category (f). Two lambda scatters with the recoil proton's length less than 1 mm were so discovered. The neutral tracks were required to be longer than 8 mm to eliminate two-prong events (elastic and inelastic beam track scatters), unless the  $K^0$  was also visible.

A considerable source of background was expected from (e) due to inner bremsstrahlung of the  $\pi^-$ . However, only two events could favorably be classified as such. Had we used a tighter acceptance criterion for "good" events, no doubt  $\Lambda \rightarrow p + \pi^- + \gamma$  events would have been a more serious source of background.

Over half of the events accepted as leptonic lambda decays were obviously such from a scanning table observation alone. The distribution in lambda production angle and in decay time for the beta decay events appeared normal within statistical errors, as did the fore-aft asymmetry of the decay.

All "failing" events were eventually classified into one of the categories described by (a) through (f) above. Contamination from all possible sources of background is believed to be zero, but is certainly less than 10%.

To study the decay interaction dynamics we have taken the expression of C. H. Albright<sup>3</sup> with the

<sup>3</sup> C. H. Albright, Phys. Rev. 115, 750 (1959).

"induced" terms and terms of higher order in  $p/m$  ( $\sim 10\%$ ) set equal to zero,<sup>4</sup> but with the coefficients of the remaining terms expanded to first order in  $p/m$ . The equation is (assuming  $C_S, T=0$ )

$$d\omega = [F(Z, E)/(2\pi)^5] p^2 (E_0 - E)^2 \xi [1 + a(\mathbf{p} \cdot \mathbf{q}/Eq) + \langle \sigma \rangle \cdot (A\mathbf{p}/E + B\mathbf{q}/q + D\mathbf{p} \times \mathbf{q}/Eq)] d\phi d\Omega_e d\Omega_\nu, \quad (2)$$

where

$\langle \sigma \rangle$  = expectation value of the lambda's spin,

$\mathbf{p}$  = electron momentum,

$E$  = electron energy,

$E_p$  = proton energy,

$\mathbf{q}$  = neutrino momentum,

$M$  = lambda mass,

$m$  = proton mass,

$\xi = (M + E_p - m) |C_V|^2 + (M + E_p + m) |C_A|^2$

$$- 2(E - q) \text{Re } C_V C_A^*,$$

$\xi a = (M - E_p + m) |C_V|^2 + (M - E_p - m) |C_A|^2$

$$+ 2(E - q) \text{Re } C_V C_A^*,$$

$\xi A = -[(E_p - m + q) |C_V|^2 + (E_p + m + q) |C_A|^2$

$$+ 2(M - E) \text{Re } C_V C_A^*],$$

$\xi B = (E_p - m + E) |C_V|^2 + (E_p + m + E) |C_A|^2$

$$- 2(M - q) \text{Re } C_V C_A^*,$$

$\xi D = -2M \text{Im } C_V C_A^*$ .

In the limit  $M = m = E_p$ , the above theory of  $\Lambda$  beta decay is precisely that of neutron beta decay.

Only events from  $K^0 \Lambda^0$  production were used for studying these angular distributions. These consisted of 22 events, 21 from this experiment, and one from an earlier experiment.<sup>5</sup> The various angular distributions are plotted in Fig. 1. The slopes (solid lines) give the best value for  $a$ ,  $PA$ ,  $PB$ , and  $PD$  as determined by the maximum likelihood method, treating each distribution as independent.  $P$  is the polarization of the lambdas. It is seen that  $D$  is consistent with zero. Therefore, time-reversal invariance is assumed to hold, and  $C_V$  and  $C_A$  are accordingly taken to be real. The intensity distribution in all variables is then determined entirely by the ratio  $k = C_A/C_V$ . Maximum use of the available information is achieved for the best determination of  $k$  by performing a likelihood fit to all of Eq. (2) as a function of  $k$ . The average weighted polarization for

<sup>4</sup> The "induced" term  $d'$  in Albright's expression plays a special role.  $C_V$  as used in the text is the customary vector coupling constant used in weak interactions. However, to zeroth order in  $p/m$  this  $C_V$  is equivalent to Albright's  $c + 2d'$ , regardless of the size of  $d'$ . Even to first order in  $p/m$ , our  $C_V$  is equivalent to Albright's  $c + 2d'$ , provided  $d'$  is small. Thus, we believe we have defined  $C_V$  in such a way as to eliminate errors of order  $p/m$  in the determination of  $C_A/C_V$  from the experimental data.

<sup>5</sup> F. S. Crawford, Jr., M. Cresti, M. L. Good, G. R. Kalbfleish, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters 1, 377 (1958).

the lambdas used was  $P = -0.91 \pm 0.10$ .<sup>6</sup> The likelihood values for  $k$  for this value of  $P$  and also for  $P$  lowered 3 standard deviations are plotted in Fig. 2. The likelihood function goes through a maximum for  $k = -1.03$ .

It is observed from Fig. 2 that  $C_A = 0$  and  $C_V = +C_A$  are ruled out with  $4 \times 10^5/1$  and  $6 \times 10^4/1$  odds, respectively, against them. Also  $C_V = 0$  is less likely than  $C_V = -C_A$  by 38/1 odds.

A plot of the proton energy spectrum in the lambda's rest frame for all 25 events is presented in Fig. 3. It is seen to be most consistent with both  $V$  and  $A$  being present.

Terms proportional to  $p/m$  and  $q/m$  in Albright's paper are  $K(\sigma) \cdot (p/m) [(p \cdot q)/E_q]$  and  $L(\sigma) \cdot (q/m) [(p \cdot q)/E_q]$ . The expectation values for  $PKp/m$  and  $PLq/m$  are found to be  $-0.64 \pm 0.64$  and  $-0.97 \pm 0.64$ , respectively. Since the errors are so large, it is felt that nothing of significance concerning these terms can be said.

For the branching ratio determination of the beta decays, only events within a prescribed fiducial volume, from "acceptable" frames, and incident beam tracks

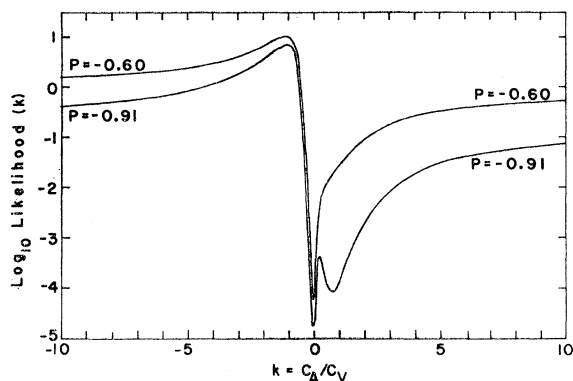


FIG. 2. The logarithm of the likelihood of  $k$ , constrained to Eq. (2), plotted versus  $k$  for 22 polarized  $\Lambda \rightarrow p + e^- + \bar{\nu}$  events for  $P = -0.91$  and  $P = -0.60$  ( $P$  lowered 3 standard deviations). The distribution of Fig. 1(a) discriminates against  $k = 0$  (slope of  $+1$ ), whereas the data of Figs. 1(c) and 1(d) combined discriminate against  $k = +1$  [large slope for 1(c) of opposite sign and a small slope for 1(d)]. This accounts for the small maximum between  $k = 0$  and  $k = +1$ , and the high ratios of the likelihood favoring  $k = -1$  to  $k = 0$  and  $k = +1$ . The surprisingly high ratios obtained are, in fact, but one standard deviation higher than the expected ratios for 22 events of this polarization.

<sup>6</sup> The values for  $\alpha^{\vec{P}}$  and the number of events at each momentum used are (momentum,  $\alpha^{\vec{P}}$ , number of events): 1030,  $0.54 \pm 0.03$ , 11; 1125,  $0.66 \pm 0.17$ , 3; 1170,  $0.67 \pm 0.10$ , 6; 1230,  $0.40 \pm 0.15$ , 1; 1275,  $0.48 \pm 0.15$ , 1. *Proceedings of the 1962 International Conference on High Energy Physics, CERN* (CERN, Geneva, 1962), University of California Lawrence Radiation Laboratory Report No. UCRL-10238 (unpublished), and unpublished results of the Alvarez group at LRL. (The sign of  $P$  is defined by the sign of  $\mathbf{p}_\pi \cdot \text{inc.} \times \mathbf{p}_\Lambda$ .) We used  $\alpha = -0.64 \pm 0.06$ , a weighted average of the following values:  $-0.62 \pm 0.07$ , J. W. Cronin and O. E. Overeth, in *Proceedings of the 1962 International Conference on High Energy Physics, CERN* (CERN, Geneva, 1962), p. 453;  $-0.67_{-0.24}^{+0.38}$ , E. F. Beall, Bruce Cork, D. Keefe, P. G. Murphy, and W. A. Wentzel, *Phys. Rev. Letters* 8, 75 (1962);  $-0.75_{-0.15}^{+0.50}$ , J. Leitner, L. Gray, E. Harth, S. Lichtman, J. Westgard *et al.*, *ibid.* 7, 264 (1961);  $-0.45 \pm 0.4$ , R. W. Birge and W. B. Fowler, *ibid.* 5, 254 (1960).

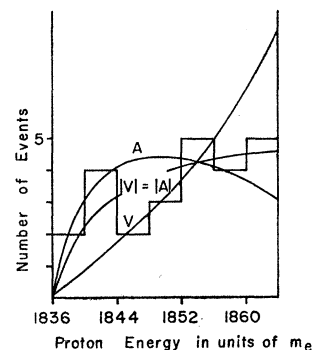


FIG. 3. A histogram plot of the proton energy spectrum in the lambda's rest frame for 25  $\Lambda \rightarrow p + e^- + \bar{\nu}$  events. The theoretical curves for pure  $V$ , pure  $A$ , and  $|C_V| = |C_A|$  were obtained from L. Egardt, *Nuovo Cimento* 27, 368 (1963).

within the beam envelope were used. Including the 7.5% correction for detection efficiency of the beta decays, there are 20.5  $\Lambda \rightarrow p + e^- + \bar{\nu}$  events and 8836  $\Lambda \rightarrow p + \pi^-$  events (excluding single  $\Lambda$ 's from  $K^0 \Sigma^0$  production) which satisfied these criteria. We obtain for a branching ratio, therefore,

$$R_e = \frac{(\Lambda \rightarrow p + e^- + \bar{\nu})}{(\Lambda \rightarrow N + \pi)} = (1.55 \pm 0.34) \times 10^{-3}.$$

This does not include one event which is ambiguous between  $\Lambda \rightarrow p + e^- + \bar{\nu}$ ,  $\Lambda \rightarrow p + \mu^- + \bar{\nu}$ , and  $\Lambda \rightarrow p + \pi^-$ ,  $\pi^- \rightarrow \mu^- + \bar{\nu}$  at "zero" length.

Combining this result for  $R_e$  with results from other experiments,<sup>7</sup> we obtain as a weighted average  $R_e = (1.07 \pm 0.13) \times 10^{-3}$ .

The lambda muonic events are so rare that all events seen are used in calculating  $R_\mu$ . An accurate count of  $\Lambda \rightarrow p + \pi^-$  is not obtainable but an estimate, accurate to 10%, is 11 500 (again not including single  $\Lambda$ 's from  $K^0 \Sigma^0$  production). Two unambiguous  $\Lambda \rightarrow p + \mu^- + \bar{\nu}$  decays were observed and including the above-mentioned ambiguous event, there were a total of seven events, ambiguous between  $\Lambda \rightarrow p + \mu^- + \bar{\nu}$  and  $\Lambda \rightarrow p + \pi^-$ ,  $\pi^- \rightarrow \mu^- + \bar{\nu}$ , the  $\pi^-$  decay taking place immediately. Since in 11 500 lambda decays we would expect a few for which the  $\pi$  decayed into a mu almost immediately ( $\sim 5$  are expected to decay within 3 mm), and since true lambda muonic events may sometimes fit the intermediate "zero"-length pion hypothesis, we set as limits (including the 16% correction)

$$R_\mu = \frac{(\Lambda \rightarrow p + \mu^- + \bar{\nu})}{(\Lambda \rightarrow N + \pi)} = \frac{2.3 \text{ to } 10.4}{(11\,500)(1.5)} = (1.3 \text{ to } 6) \times 10^{-4}.$$

The true number is presumed rather toward the lower end of the range, since most muonic  $\Lambda$  decays will not fit the  $\pi \rightarrow \mu$  decay hypothesis.

An improved value for  $R_\mu$  may be estimated by considering the world sample of reported unambiguous

<sup>7</sup> The weighted average of  $R_e$  was obtained by combining the results of this experiment with the experiments of Ely *et al.* (Ref. 1); Crawford *et al.* (Ref. 5); Joseph Keren, *Phys. Rev.* 133, B457 (1964); and B. Aubert *et al.*, *Proceedings of Aix-en-Provence International Conference* (Centre d'Etudes Nucleaires de Saclay, Saclay, France, 1961), Vol. 1, p. 197.

lambda muonic events for which the muon stops in the chamber, and making the necessary corrections for detection efficiency. We are aware of three such events having been observed in hydrogen.<sup>8</sup> For a weighted average of  $R_\mu$  we obtain<sup>9</sup>

$$R_\mu = (1.3 \pm 0.7) \times 10^{-4}.$$

Muon-electron universality predicts  $R_e/R_\mu = 6.2$  and hence  $R_\mu = 1.7 \times 10^{-4}$ , which is consistent with both of the above  $R_\mu$  determinations.

Comparing our results with those of Cabibbo,<sup>10</sup> we find a remarkable agreement. Using  $R_e = (1.07 \pm 0.13)$

<sup>8</sup> The detection efficiency for stopping muons from lambda muonic decays depends on the liquid used in the chamber, the size of the chamber, and the momentum of the lambdas. Monte Carlo calculations are used for estimating these efficiencies. For the reported lambda muonic decays we used the curves prepared by W. E. Humphrey, J. Kirz, A. H. Rosenfeld, and J. Leitner, *Proceedings of the 1962 International Conference on High Energy Physics, CERN* (CERN, Geneva, 1962), p. 442. The reported events, their respective detection efficiency, and sample size of observed lambda decays are: M. H. Alston, J. Kirz, J. Neufeld, F. T. Solmitz, and P. G. Wohlmuth, UCR-10926, 1963 (unpublished), 23%, 30 000; M. L. Good and V. G. Lind, *Phys. Rev. Letters* **9**, 518 (1962), 28%, 11 500; F. Eisler, J. M. Gaillard, J. Keren, M. Schwartz, and S. Wolf, *ibid.* **7**, 136 (1961), 24%, 900.

<sup>9</sup> Two (not completely unambiguous)  $\Lambda_\mu$  events in freon in an effective sample of 19 700 lambdas have been observed. The experimenters deduce that  $R_\mu \leq 4.5 \times 10^{-4}$  at the 5% significance level. A. Kernan, W. M. Powell, C. L. Sandler, W. L. Knight, and F. R. Stannard, *Phys. Rev.* **133**, B1271 (1964).

<sup>10</sup> Nicola Cabibbo, *Phys. Rev. Letters* **10**, 531 (1963).

$\times 10^{-3}$ ,  $\sin\theta = 0.206$ <sup>11</sup> (Sakurai's correction to  $\theta = 0.26$  reported by Cabibbo) we calculate  $|k| = 1.09 \pm 0.09$ .

$$\Gamma(\Lambda \rightarrow p + e^- + \bar{\nu}) = 2.15 \times 10^7 \text{ sec}^{-1} \sin^2\theta (1 + 3|k|^2).$$

This is certainly consistent within errors with  $k$  as measured above. At  $e^{-1/2}$  times the maximum in the likelihood function we obtain as the errors on  $k$ ,  $-0.70$ , and  $+0.34$ .

Finally, this experiment does not exclude the possibility of a mixture of  $S$  and  $T$  instead of  $V$  and  $A$  as the interaction currents. If  $S$  and  $T$  were the correct currents, a likelihood calculation favors  $C_T = -0.50C_S$ .

Our results are consistent with the conclusions of the experiment of C. Baglin *et al.*<sup>12</sup> They rule out pure  $V$  but do not decide between pure  $A$  and  $|C_V| = |C_A|$ .

The cooperation of L. W. Alvarez and particularly the cooperation and assistance of Frank S. Crawford, Jr. and his co-workers are greatly appreciated. Furthermore, assistance from the scanning, measuring, and computing staff at both LRL and the University of Wisconsin as well as the many graduate students who worked on various phases of the experiment is gratefully acknowledged.

Dr. Bunji Sakita is thanked for enlightening discussions on the theoretical interpretation.

<sup>11</sup> J. J. Sakurai, *Phys. Rev. Letters* **12**, 79 (1964).

<sup>12</sup> C. Baglin, V. Brisson, A. Rousset, J. Six, H. H. Bingham *et al.*, *Phys. Letters* **6**, 186 (1963).

## Macroscopic Bodies in Quantum Theory\*

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It is shown that if the wave function of a massive body is  $\psi = \sum c_n \psi_n$ , where the  $\psi_n$  are macroscopically distinguishable states, then the observation of interferences between the various  $\psi_n$  requires inconceivable laboratory conditions (e.g., the experiment may last longer than the lifetime of the universe). It is therefore proposed to interpret  $\sum c_n \psi_n$  as a mixture of states, and not as a superposition. This new interpretation of wave functions is consistent with experience and is free from the paradoxical features of the "orthodox" measurement theory.

IT may seem strange that more than thirty years after von Neumann's classic work,<sup>1</sup> the problem of measurement in quantum theory is not yet considered as settled.<sup>2</sup> The difficulty can easily be illustrated as follows. Suppose we have an instrument designed so as to

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<sup>1</sup> J. von Neumann, *Mathematische Grundlagen der Quantenmechanik* (Springer-Verlag, Berlin, 1932).

<sup>2</sup> We quote only a few recent papers in which many further references may be found: S. Amai, *Progr. Theoret. Phys.* (Kyoto) **30**, 550 (1963); H. Margenau, *Ann. Phys.* (N. Y.) **23**, 469 (1963); A. Shimony, *Am. J. Phys.* **31**, 755 (1963); E. P. Wigner, *ibid.* **31**, 6 (1963); M. M. Yanase, *ibid.* **32**, 208 (1964).

measure a dynamical variable  $A$  belonging to a quantum system  $S$ . If  $S$  is initially in an eigenstate  $\phi_i$  of  $A$ , then the pointer of the instrument will show the corresponding eigenvalue  $a_i$ . If  $S$  is initially in an eigenstate  $\phi_j$ , the pointer will show the eigenvalue  $a_j$  (we suppose  $a_j \neq a_i$ ). Now, if  $S$  is initially in the state  $\phi = 2^{-1/2}(\phi_i + \phi_j)$ , then the pointer will finally indicate either  $a_i$  or  $a_j$ , with equal probabilities. It will *not* be partly at  $a_i$  and partly at  $a_j$ , even though the initial state of  $S$  was a superposition of  $\phi_i$  and  $\phi_j$ . In other words, *the superposition principle is violated in a measurement process.*

Quite generally, von Neumann has shown that interactions can be constructed such that, if the initial state of  $S$  is  $\phi = \sum c_n \phi_n$ , and if the initial state of the instru-